CONSERVED NOETHER CURRENTS IN STOCHASTIC QUANTIZATION

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A general procedure for constructing Noether conserved currents in the stochastic quantization scheme corresponding to symmetries of the equilibrium theory is proposed. Two different regularizations – the Breit–Gupta–Zaks stochastic time regularization and a new supersymmetric regularization – are employed, and the origin of chiral anomalies is exhibited in this framework.

1. By its introduction [1] the stochastic quantization scheme (SQS) was intended to give an alternative way of quantization of field theory models as well as to provide new invariant regularizations [2] which presumably respect simultaneously all symmetries of the underlying models.

In usual field theory symmetries of the classical action correspond via the Noether theorem to conserved currents which, upon quantization, yield Ward identities for the correlation functions of the quantum fields. Unlike this, the original formulation of SQS [1] in terms of the Langevin equations is not based on an action principle and, therefore, the symmetries of the SQS averages were not expressed in terms of Ward-like identities.

In the present letter we propose a general procedure for deriving SQS Noether conserved currents within the superspace formulation [3,4] of SQS. In fact, these SQS currents appear as conserved supercurrents in the auxiliary $(D+1|2)$-dimensional superspace [with coordinates $z=(x,\tau;\theta,\bar\theta), x\in \mathrm{R}^D$]. After introducing a new invariant regularization respecting the stochastic supersymmetry, we show that only the lowest components (in the θ-expansion) of the SQS Noether currents survive in the equilibrium limit and yield equilibrium Ward identities coinciding with the standard ones. The same result is achieved by employing the Breit–Gupta–Zaks (BGZ) regularization [2] in the superspace approach.

The question of anomalous chiral symmetries in this framework is also considered. Although there exists a classical" Noether conserved chiral supercurrent in $(D+1|2)$ dimensions, upon quantization it precisely yields the correct standard (covariant) chiral anomalies in the equilibrium limit. This phenomenon is here understood from the point of view of both regularizations – the BGZ and the supersymmetric one.

2. Let us begin with briefly recalling the superspace formulation [3,4] of SQS. In short-hand notations the Langevin equations for a general D-dimensional field theory model with a classical action $S[\varphi]=\int \mathrm{d}^D x\, \mathcal{L}(\varphi)$ read

$$\partial_\tau \varphi_\eta = -\mathcal{K}\,\delta S/\delta\varphi|_{\varphi=\varphi_\eta} + \eta, \quad \langle \eta(x,\tau)\eta(x',\tau')\rangle = 2\mathcal{K}\,\delta(\tau-\tau')\delta^{(D)}(x-x'), \tag{1}$$

where $\mathcal{K}\equiv\mathcal{K}[\partial_x]$ denotes an appropriate differential operator (in x) which ensures the positiveness of the corresponding Fokker–Planck hamiltonian [5,2]. More generally, $\mathcal{K}$ may be chosen to depend functionally on φ_η and in this case (1) is changed as follows [5,6]:

$$\partial_\tau \varphi_\eta = -\mathcal{K}\,\delta S/\delta\varphi|_{\varphi=\varphi_\eta} + \mathcal{K}\,\eta_1 - \mathrm{i}\eta_2, \tag{1'}$$

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$$\langle\eta_1(x,\tau)\eta_2(x',\tau')\rangle = -\mathrm{i}\delta(\tau-\tau')\delta^{(D)}(x-x'),\quad \langle\eta_{1,2}\eta_{1,2}\rangle = 0, \qquad (1'\ \text{cont'd})$$

where two independent random sources are introduced.

Choosing initial conditions for (1) $\varphi_\eta(x,\tau=-\infty)=0$, the generating functional of the SQS averages of solutions φ_η to (1),

$$Z[h] = \exp(-\tfrac{1}{2}x\ \mathrm{Tr}\ln\mathcal{K})\int\mathcal{D}\eta\,\exp\left(\int \mathrm{d}^Dx\,\mathrm{d}\tau[-\tfrac{1}{4}\eta\mathcal{K}^{-1}\eta + h\varphi_\eta]\right) \qquad (2)$$

($x=\pm1$ for φ,η being bosons or fermions respectively), turns out to be (formally) equal to the generating functional of the correlation functions of the following supersymmetric field theory in $(D+1|2)$-dimensional superspace:

$$Z[h] = \mathcal{Z}[H] = \int\mathcal{D}\Phi\,\exp\left(\int \mathrm{d}^{D+1|2}z\,(-\Sigma[\Phi]+H\Phi)\right), \qquad (3)$$

$$\Sigma[\Phi] = \int \mathrm{d}^{D+1|2}z\,[\tfrac{1}{2}\bar{\mathrm{D}}\Phi\mathcal{K}^{-1}\mathrm{D}\Phi - \tfrac{1}{2}\mathrm{D}\Phi\mathcal{K}^{-1}\bar{\mathrm{D}}\Phi - \mathrm{i}\mathcal{L}(\Phi)], \qquad (4)$$

There

$$\Phi(z) = \varphi(x,\tau) + \bar\theta\chi(x,\tau) + \bar\chi(x,\tau)\theta + \bar\theta\theta F(x,\tau),\quad H(z) = \bar\theta\theta h(x,\tau),$$

$$\mathrm{D} = \partial/\partial\theta,\quad \bar{\mathrm{D}} = \partial/\partial\bar\theta - \mathrm{i}\theta\partial_\tau,\quad \{\mathrm{D},\bar{\mathrm{D}}\} = -\mathrm{i}\partial_\tau. \qquad (5)$$

The form of the grassmannian derivatives D, $\bar{\mathrm{D}}$ corresponds to a chiral representation of the supersymmetry transformations with generators

$$Q = \partial/\partial\theta + \tfrac{1}{2}\mathrm{i}\bar\theta\partial_\tau,\quad \bar{Q} = \partial/\partial\bar\theta + \tfrac{1}{2}\mathrm{i}\,\theta\partial_\tau,\quad \{Q,\bar{Q}\} = \mathrm{i}\partial_\tau,$$

$$\delta\tilde\Phi(z) = (\epsilon Q + \bar\epsilon\bar{Q})\tilde\Phi(z) = \tilde\Phi(x,\tau+\tfrac{1}{2}\mathrm{i}(\bar\epsilon\theta-\bar\theta\epsilon),\theta+\epsilon,\bar\theta+\bar\epsilon),\quad \tilde\Phi(z) \equiv \exp\{-\tfrac{1}{2}\mathrm{i}\,\bar\theta\theta\partial_\tau\}\Phi(z). \qquad (6)$$

It can be straightforwardly checked that formulae (3), (4) remain valid also for the more general SQS (1′) where $\mathcal{K}=\mathcal{K}[\Phi;\partial_x]$ now explicitly depends on the superfield Φ:

$$Z'[h] = \int\mathcal{D}\eta_1\,\mathcal{D}\eta_2\,\exp\left(\int\mathrm{d}^Dx\,\mathrm{d}\tau[\mathrm{i}\eta_1\eta_2 + h\varphi_\eta]\right) = \mathcal{Z}'[H],\quad \Sigma'[\Phi] = \Sigma[\Phi] + \Sigma_1[\Phi]. \qquad (3', 4')$$

In what follows we shall not need the explicit form of $\Sigma_1[\Phi]$ since it vanishes for background (not quantized) fields.

Also, in this case we have $\eta_2(x,\tau) = F(x,\tau)$, where F is the highest component in the θ-expansion of Φ (5). Let us note that eqs. (3), (3′) are particular realizations of the Nicolai map [7].

It is easy to see (cf. ref. [4]) that the superspace action $\Sigma[\Phi]$ (4) respects all symmetries G of the original D-dimensional action $S[\varphi]$ [provided $\mathcal{K}$ was chosen in such a way that (1), (1′) remain G-covariant]. Then we can apply the standard Noether procedure to find conserved currents in the $(D+1|2)$-dimensional supersymmetric theory which will precisely represent the SQS analogues of the usual Noether currents in D dimensions.

Indeed, the Noether theorem implies, due to the G-invariance of (3), the existence of the following conserved supercurrent:

$$\bar{\mathrm{D}}J_\theta[\Phi] + \mathrm{D}J_{\bar\theta}[\Phi] + \partial_\mu J_\mu[\Phi] = 0, \qquad (7)$$

$$J_\theta[\Phi] = \mathrm{i}\delta\Phi(\mathcal{K}^{-1}\mathrm{D}\Phi),\quad J_{\bar\theta}[\Phi] = -\mathrm{i}\delta\Phi(\mathcal{K}^{-1}\mathrm{D}\Phi), \qquad (8a)$$

$$J_\mu[\Phi] = j_\mu[\Phi] - \tfrac{1}{2}\mathrm{i}(\mathrm{D}\Phi)(\delta\mathcal{K}^{-1}[B]/\delta B_\mu|_{B=0})(\bar{\mathrm{D}}\Phi) + \tfrac{1}{2}\mathrm{i}(\bar{\mathrm{D}}\Phi)(\delta\mathcal{K}^{-1}[B]/\delta B_\mu|_{B=0})(\mathrm{D}\Phi), \qquad (8b)$$

$$j_\mu[\Phi] = \delta\Phi(\partial\mathcal{L}/\partial(\partial_\mu\Phi) - \mathcal{R}_\mu), \tag{8c}$$

where $\delta\Phi$, $\delta\mathcal{L} = \partial^\mu\mathcal{R}_\mu$ denote the standard variations under the corresponding G-transformations and $\mathcal{K}[B] \equiv \mathcal{K}[\partial_x + B]$ is the gauge-covariant counterpart of $\mathcal{K}[\partial_x]$ with B_μ being an auxiliary G-gauge potential.

3. In proceeding with the quantization of the supercurrents (7), (8) we shall use the proper-time representation of the free Φ-propagator. This representation offers a possibility to introduce a new ultraviolet regularization manifestly preserving all symmetries of the original theory defined by $S[\varphi]$ as well as preserving the stochastic supersymmetry. We have

$$\langle\Phi(z)\Phi(z')\rangle^{(0)}_{\text{reg}} = \mathrm{i}\mathcal{K}\int_0^\infty \mathrm{d}\alpha\, \rho_\Lambda(\alpha)\exp(-\alpha\mathcal{K}S'')(x,x')\exp(-\mathrm{i}\alpha[\mathrm{D},\bar{\mathrm{D}}])\,\delta(\tau-\tau')\delta^{(2)}(\theta-\theta'), \tag{9}$$

where

$$S'' \equiv \delta^2 S/\delta\varphi^2|_{\varphi=0}, \quad \delta^{(2)}(\theta-\theta') \equiv (\bar\theta-\bar\theta')(\theta-\theta'),$$

and the regularizing function $\rho_\Lambda(\alpha)$ obeys the properties

$$\lim_{\Lambda\to\infty}\rho_\Lambda(\alpha) = 1, \qquad (\mathrm{d}^k/\mathrm{d}\alpha^k)\rho_\Lambda(\alpha)|_{\alpha=0} = 0, \quad k = 0, 1, ..., L \quad (\alpha \geqslant 0), \tag{10}$$

L being an appropriate integer depending on the spacetime dimension D. A particular choice of $\rho_\Lambda(\alpha)$ is

$$\rho_\Lambda(\alpha) = 1 - \exp(-\Lambda\alpha)\left(\sum_{k=0}^{L}\frac{1}{k!}(\Lambda\alpha)^k\right). \tag{10'}$$

With the regularized propagator (9), all eventual ultraviolet divergences in the diagram expansion of $\mathcal{Z}[H]$ (3) which manifest themselves as singularities $\mathrm{O}(\alpha^{-k})$, $k \geqslant 1$, in the proper-time integrals entering the diagrams are regulated by $\rho_\Lambda(\alpha)$.

It is easy to show that, due to the τ-translation invariance and the supersymmetry (6), all equal-τ and equal-$\theta,\bar\theta$ correlation functions given by (3) are in fact τ- and θ-independent and, therefore, they are equal to their equilibrium limits:

$$\begin{aligned}\langle\Phi(x_1,\tau;\theta,\bar\theta)\,...\,\Phi(x_N,\tau;\theta,\bar\theta)\rangle &= \lim_{\tau\to\infty}\langle\varphi(x_1,\tau)\,...\,\varphi(x_N,\tau)\rangle = \langle\varphi(x_1)\,...\,\varphi(x_N)\rangle \\ &\equiv (\delta^N/\delta j^N)\int\mathcal{D}\varphi\exp\left(\int\mathrm{d}^D x\,[-\mathcal{L}(\varphi) + j\varphi]\right)\Big|_{j=0}.\end{aligned} \tag{11}$$

Property (11) will be important in analyzing the Ward identities corresponding to the Noether conserved supercurrents (7) in the next section.

4. Now we shall apply the general scheme described in sections 2, 3 to the SQS for fermions $\psi(x,\tau)$ in an external background U(n)-gauge field $A_\mu(x)$. In this case eqs. (1), (3), (4) are specialized with

$$S[\psi,\tilde\psi] = \int\mathrm{d}^D x\,\tilde\psi(x)(m + \mathrm{i}\not{\nabla})\psi(x), \quad \mathcal{K} = m - \mathrm{i}\not{\nabla} \quad \text{(cf. refs. [6,8,2])},$$

$$\mathcal{Z}[H,\tilde H] = \int\mathcal{D}\Psi\,\mathcal{D}\tilde\Psi\exp\left(-\Sigma[\Psi,\tilde\Psi] + \int\mathrm{d}^{D+1|2}z\,(\tilde H\Psi + \tilde\Psi H)\right), \tag{12a}$$

$$\Sigma[\Psi,\widetilde{\Psi}] = \int \mathrm{d}^{D+1|2}z\ \{\mathrm{D}\widetilde{\Psi}(m - \mathrm{i}\nabla\!\!\!\!/)^{-1}\bar{\mathrm{D}}\Psi - \bar{\mathrm{D}}\Psi(m - \mathrm{i}\nabla\!\!\!\!/)^{-1}\mathrm{D}\Psi + \mathrm{i}\widetilde{\Psi}(m + \mathrm{i}\nabla\!\!\!\!/)\Psi\} \quad \text{(cf. also ref. [9])}, \tag{12b}$$

with the following notations:

$$\nabla\!\!\!\!/ = \gamma_\mu[\partial_\mu + \mathrm{i}A_\mu(x)], \quad \{\gamma_\mu,\gamma_\nu\} = -2\delta_{\mu\nu}, \quad A_\mu(x) = T^a A^a_\mu(x),$$

$$\Psi(z) = \psi(x,\tau) + \bar{\theta}\omega_1(x,\tau) + \omega_2(x,\tau)\theta + \bar{\theta}\theta\Omega(x,\tau),$$

$$\widetilde{\Psi}(z) = \widetilde{\psi}(x,\tau) + \bar{\theta}\widetilde{\omega}_2(x,\tau) + \widetilde{\omega}_1(x,\tau)\theta + \bar{\theta}\theta\Omega(x,\tau), \tag{13}$$

where the tilde corresponds to Dirac conjugation and $\{T^a\}$ ($a = 0, 1, \ldots, n^2 - 1$) form a hermitian basis of the U(n)-generators.

The classical conserved Noether supercurrent corresponding to the vector U(n) invariance of (12) reads

$$J^a_\theta(z) = \mathrm{i}[\mathrm{D}\widetilde{\Psi}(m - \mathrm{i}\nabla\!\!\!\!/)^{-1}]T^a\Psi + \mathrm{i}\widetilde{\Psi}T^a[(m - \mathrm{i}\nabla\!\!\!\!/)^{-1}\mathrm{D}\Psi], \tag{14a}$$

$$J^a_{\bar{\theta}}(z) = -\mathrm{i}[\bar{\mathrm{D}}\widetilde{\Psi}(m - \mathrm{i}\nabla\!\!\!\!/)^{-1}]T^a\Psi - \mathrm{i}\widetilde{\Psi}T^a[(m - \mathrm{i}\nabla\!\!\!\!/)^{-1}\bar{\mathrm{D}}\Psi] \tag{14b}$$

$$J^a_\mu(z) = \widetilde{\Psi}T^a(-\mathrm{i}\gamma_\mu)\Psi + [\bar{\mathrm{D}}\widetilde{\Psi}(m - \mathrm{i}\nabla\!\!\!\!/)^{-1}]T^a\gamma_\mu[(m - \mathrm{i}\nabla\!\!\!\!/)^{-1}\mathrm{D}\Psi] - [\mathrm{D}\widetilde{\Psi}(m - \mathrm{i}\nabla\!\!\!\!/)^{-1}]T^a\gamma_\mu[(m - \mathrm{i}\nabla\!\!\!\!/)^{-1}\bar{\mathrm{D}}\Psi], \tag{14c}$$

$$\bar{\mathrm{D}}J^a_\theta + \mathrm{D}J^a_{\bar{\theta}} + \nabla^{ab}_\mu J^b_\mu = 0. \tag{15}$$

Likewise, the axial U(n) symmetry of (12b) when $m = 0$ implies the existence of a conserved classical axial Noether supercurrent (here D is even):

$$J^{(D+1)a}_\theta(z) = \mathrm{i}\widetilde{\Psi}T^a\gamma^{(D+1)}[(m - \mathrm{i}\nabla\!\!\!\!/)^{-1}\mathrm{D}\Psi] - \mathrm{i}[\mathrm{D}\widetilde{\Psi}(m - \mathrm{i}\nabla\!\!\!\!/)^{-1}]T^a\gamma^{(D+1)}\Psi, \tag{16a}$$

$$J^{(D+1)a}_{\bar{\theta}}(z) = -\mathrm{i}\widetilde{\Psi}T^a\gamma^{(D+1)}[(m - \mathrm{i}\nabla\!\!\!\!/)^{-1}\bar{\mathrm{D}}\Psi] + \mathrm{i}[\bar{\mathrm{D}}\widetilde{\Psi}(m - \mathrm{i}\nabla\!\!\!\!/)^{-1}]T^a\gamma^{(D+1)}\Psi, \tag{16b}$$

$$J^{(D+1)a}_\mu(z) = \widetilde{\Psi}T^a(-\mathrm{i}\gamma_\mu)\gamma^{(D+1)}\Psi + [\bar{\mathrm{D}}\widetilde{\Psi}(m - \mathrm{i}\nabla\!\!\!\!/)^{-1}]T^a\gamma_\mu\gamma^{(D+1)}[(m - \mathrm{i}\nabla\!\!\!\!/)^{-1}\mathrm{D}\Psi]$$

$$- [\mathrm{D}\widetilde{\Psi}(m - \mathrm{i}\nabla\!\!\!\!/)^{-1}]T^a\gamma_\mu\gamma^{(D+1)}[(m - \mathrm{i}\nabla\!\!\!\!/)^{-1}\bar{\mathrm{D}}\Psi]. \tag{16c}$$

The classical conservation law for (16) reads

$$\bar{\mathrm{D}}J^{(D+1)a}_\theta + \mathrm{D}J^{(D+1)a}_{\bar{\theta}} + \nabla^{ab}_\mu J^{(D+1)b}_\mu = -2m\widetilde{\Psi}T^a\gamma^{(D+1)}\Psi$$

$$- 2\mathrm{i}m[\mathrm{D}\widetilde{\Psi}(m - \mathrm{i}\nabla\!\!\!\!/)^{-1}]T^a\gamma^{(D+1)}[(m - \mathrm{i}\nabla\!\!\!\!/)^{-1}\bar{\mathrm{D}}\Psi] + 2\mathrm{i}m[\bar{\mathrm{D}}\widetilde{\Psi}(m - \mathrm{i}\nabla\!\!\!\!/)^{-1}]T^a\gamma^{(D+1)}[(m - \mathrm{i}\nabla\!\!\!\!/)^{-1}\mathrm{D}\Psi]. \tag{17}$$

Due to the general property (11) only the lowest components in the θ-expansion [cf. (13)] of eqs. (15) and (17) carry physical information in the quantum averages.

We shall evaluate the quantum expectation values of eqs. (15), (17) by using two different regularizations of the fermion propagator. The first one is the supersymmetric regularization (9) giving

$$\langle\Psi(z)\widetilde{\Psi}(z')\rangle^{\text{susy}}_{\text{reg}} = (m - \mathrm{i}\nabla\!\!\!\!/)\{-\tfrac{1}{2}\mathrm{i}\,\rho_\Lambda(|\tau - \tau'|)\exp[-(m^2 + \nabla\!\!\!\!/^2)|\tau - \tau'|](x,x')\delta^{(2)}(\theta - \theta')$$

$$+ \tfrac{1}{2}[\mathrm{D},\bar{\mathrm{D}}]\delta^{(2)}(\theta - \theta')\int_{|\tau - \tau'|}^{\infty}\mathrm{d}\alpha\,\rho_\Lambda(\alpha)\exp[-\alpha(m^2 + \nabla\!\!\!\!/^2)](x,x'). \tag{18}$$

The second one is the BGZ regularization [2] which amounts to the following change of the grassmannian derivatives (5):

$$\mathrm{D} = \partial/\partial\theta, \quad \bar{\mathrm{D}} = \hat{\delta}_\Lambda\partial/\partial\bar{\theta} - \mathrm{i}\theta\partial_\tau. \tag{5'}$$

The BGZ-regulated propagator (after taking the Fourier transform with respect to $\tau-\tau'$) reads

$$\langle\Psi\bar{\Psi}\rangle_{\mathrm{BGZ}}(x,\theta,\bar{\theta};x',\theta',\bar{\theta}';\omega)=(m-\mathrm{i}\not{\nabla})\{\tilde{\delta}_\Lambda(\omega)(m^2+\not{\nabla}^2+\omega^2)^{-1}+\tfrac{1}{2}(\bar{\theta}'-\bar{\theta})\theta[\omega-\mathrm{i}(m^2+\not{\nabla}^2)]^{-1}$$
$$+\tfrac{1}{2}(\bar{\theta}'-\bar{\theta})\theta'[\omega+\mathrm{i}(m^2+\not{\nabla}^2)]^{-1}\}(x,x'). \tag{19}$$

In eq. (19) $\tilde{\delta}_\Lambda(\omega)$ denotes the Fourier transform of the BGZ-regularized δ-function $\delta_\Lambda(\tau-\tau')$ entering eq. (1) and obeying the properties

$$\lim_{\Lambda\to\infty}\delta_\Lambda(\alpha)=\delta(\alpha),\quad \delta_\Lambda(-\alpha)=\delta_\Lambda(\alpha),\quad (\mathrm{d}^k/\mathrm{d}\alpha^k)\delta_\Lambda(\alpha)|_{\alpha=0}=0,\quad k=0,1,\ldots,L', \tag{20}$$

where L' is an appropriate integer depending on D. In eq. (5') $\hat{\delta}_\Lambda$ denotes an integral operator with kernel $\delta_\Lambda(\tau-\tau')$. Comparing (20) and (10) we see that one can take

$$2\delta_\Lambda(\alpha)=(\mathrm{d}/\mathrm{d}\alpha)\rho_\Lambda(\alpha)\quad(\alpha\geqslant 0). \tag{21}$$

Although the above regularizations are completely different [note from (19) that the BGZ regularization explicitly breaks supersymmetry (6)] one can show that in the equilibrium limit they yield identical results for the regularized diagrams of (3). In particular, this holds for the expectation values of (15) and (17).

Since the regularization (18) explicitly preserves supersymmetry, then accounting for (11) we get:

$$\mathrm{D}\langle J^a_{\bar{\theta}}(z)\rangle=\bar{\mathrm{D}}\langle J^a_\theta(z)\rangle=0,\quad \mathrm{D}\langle J^{(D+1)a}_{\bar{\theta}}(z)\rangle=\bar{\mathrm{D}}\langle J^{(D+1)a}_\theta(z)\rangle=0. \tag{22}$$

Next, from (10) and using the algebra (5) of the D, $\bar{\mathrm{D}}$-derivatives, we find that only the lowest superfield component of the first terms on the RHS of (14c), (16c) survive in the expectation values:

$$\langle J^a_\mu(z)\rangle=\langle\tilde{\psi}(x,\tau)T^a(-\mathrm{i}\gamma_\mu)\psi(x,\tau)\rangle=\int_0^\infty\mathrm{d}\alpha\,\rho_\Lambda(\alpha)\,\mathrm{tr}[T^a\gamma_\mu(\mathrm{i}m+\not{\nabla})\exp[-\alpha(m^2+\not{\nabla}^2)](x,x)], \tag{23}$$

$$\langle J^{(D+1)a}_\mu(z)\rangle=\langle\tilde{\psi}(x,\tau)T^a(-\mathrm{i}\gamma_\mu)\gamma^{(D+1)}\psi(x,\tau)\rangle$$
$$=\int_0^\infty\mathrm{d}\alpha\,\rho_\Lambda(\alpha)\,\mathrm{tr}[T^a\gamma_\mu\gamma^{(D+1)}(\mathrm{i}m+\not{\nabla})\exp[-\alpha(m^2+\not{\nabla}^2)](x,x)]. \tag{24}$$

Expressions (23), (24) exactly coincide with those obtained in ref. [10] using the BGZ regularization in the Langevin approach, provided $\rho_\Lambda(\alpha)$ and $\delta_\Lambda(\alpha)$ are connected by (21) [this can also be directly checked using (19)]. As a consequence, as it was already demonstrated in refs. [10,11], we get quantum conservation of the vector U(n) symmetry:

$$\nabla^{ab}_\mu\langle\tilde{\psi}(x,\tau)T^b(-\mathrm{i}\gamma_\mu)\psi(x,\tau)\rangle=0,\quad\text{i.e.}\quad\langle\bar{\mathrm{D}}J^a_\theta(z)+\mathrm{D}J^a_{\bar{\theta}}(z)+\nabla^{ab}_\mu J^b_\mu(z)\rangle=0, \tag{25}$$

and, accordingly, we obtain the standard U(n)-axial anomaly:

$$\lim_{m\to 0}\nabla^{ab}_\mu\langle\tilde{\psi}(x,\tau)T^b(-\mathrm{i}\gamma_\mu)\gamma^{(D+1)}\psi(x,\tau)\rangle=-2\mathcal{A}^a(x)+2\,\mathrm{tr}[T^a\gamma^{(D+1)}\Pi_0(x,x)],$$

$$\mathcal{A}^a(x)=[(D/2)!(4\pi)^{D/2}]^{-1}\,\mathrm{tr}[T^aF_{\mu_1\mu_2}(x)\ldots F_{\mu_{D-1}\mu_D}(x)]\,\epsilon_{\mu_1\ldots\mu_D},$$

i.e.

$$\lim_{m\to 0}\langle\bar{\mathrm{D}}J^{(D+1)a}_\theta(z)+\mathrm{D}J^{(D+1)a}_{\bar{\theta}}(z)+\nabla^{ab}_\mu J^{(D+1)b}_\mu(z)\rangle=-2\mathcal{A}^a(x)+2\,\mathrm{tr}[T^a\gamma^{(D+1)}\Pi^{\not{\nabla}}_0(x,x)], \tag{26}$$

where $\Pi^{\not{\nabla}}_0$ denotes the kernel of the zero-mode projector of $\not{\nabla}$ and eq. (22) was accounted for.

Now, let us check eq. (26) with the BGZ regularization (5′), (19). In this case (22) does not hold and, therefore, we consider the lowest component of the full expression in (17), (16)

$$\nabla_\mu^{ab}(\tilde{\psi}T^b(-\mathrm{i}\gamma_\mu)\gamma^{(D+1)}\psi) + 2\mathrm{i}[\tilde{\omega}_1(m-\mathrm{i}\nabla\!\!\!/)^{-1}]T^a\gamma^{(D+1)}\hat{\delta}_\Lambda\omega_1 - 2\mathrm{i}[\hat{\delta}_\Lambda\tilde{\omega}_2(m-\mathrm{i}\nabla\!\!\!/)^{-1}]T^a\gamma^{(D+1)}\omega_2$$
$$-\mathrm{i}[(2\hat{\delta}_\Lambda\tilde{\Omega}-\mathrm{i}\partial_\tau\tilde{\psi})(m-\mathrm{i}\nabla\!\!\!/)^{-1}]T^a\gamma^{(D+1)}\psi + \mathrm{i}\tilde{\psi}T^a\gamma^{(D+1)}[(m-\mathrm{i}\nabla\!\!\!/)^{-1}(2\hat{\delta}_\Lambda\Omega-\mathrm{i}\partial_\tau\psi)]$$
$$= -2m\tilde{\psi}T^a\gamma^{(D+1)}\psi. \tag{27}$$

Performing the quantum average, the last two terms on the LHS of (27) cancel and we obtain

$$\nabla_\mu^{ab}\langle\tilde{\psi}T^a(-\mathrm{i}\gamma_\mu)\gamma^{(D+1)}\psi\rangle = -2\mathrm{i}\langle[\tilde{\omega}_1(m-\mathrm{i}\nabla\!\!\!/)^{-1}]T^a\gamma^{(D+1)}\hat{\delta}_\Lambda\omega_1\rangle$$
$$+ 2\mathrm{i}\langle[\hat{\delta}_\Lambda\tilde{\omega}_2(m-\mathrm{i}\nabla\!\!\!/)^{-1}]T^a\gamma^{(D+1)}\omega_2\rangle - 2m\langle\tilde{\psi}T^a\gamma^{(D+1)}\psi\rangle. \tag{28}$$

The remarkable feature of eq. (28) is that the $\omega_{1,2}$-terms precisely yield the standard covariant U(n)-anomaly. Indeed, computing the averages of the $\omega_{1,2}$-terms in (28) from (19) we get

$$\lim_{\Lambda\to\infty} -4\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}\,\mathrm{tr}[T^a\gamma^{(D+1)}\tilde{\delta}_\Lambda(\omega)(m^2+\nabla\!\!\!/^2+\mathrm{i}\omega)^{-1}(x,x)]$$
$$= \lim_{\Lambda\to\infty} -4\int_0^{\infty}\mathrm{d}\alpha\,\delta_\Lambda(\alpha)\,\mathrm{tr}[T^a\gamma^{(D+1)}\exp[-\alpha(m^2+\nabla\!\!\!/^2)](x,x)] = -2\mathcal{A}^a(x).$$

Thus, once again the anomalous Ward identity (26) is derived.

As it was already stressed in ref. [11], there is *no* contradiction between the following two properties of SQS:

(i) Manifest preservation of chiral-gauge symmetries in the SQS averages [either given by the Langevin (2) or the superspace (3) generating functional] through invariant stochastic regularizations (either the BGZ regularization (18) or the supersymmetric one (9));

(ii) Appearance of the correct chiral anomalies (26).

The reason for the above fact is as follows. The present chiral symmetry-preserving regularizations of the SQS averages do *not* simultaneously regularize the quantum fermion effective action

$$S_{\mathrm{eff}} = \mathrm{Tr}\ln[-\mathrm{i}\nabla\!\!\!/] \tag{29}$$

[in particular, for the choice (10′), (21) they give in the equilibrium limit regularization of the fermion propagator (18), (19) with higher covariant derivatives]. However, S_{eff} (29), whose chiral non-invariance is responsible for the anomalous chiral Ward identities, does *not* enter SQS at all since (29) can never be represented as a SQS average of a certain functional of the solutions $\overset{(\sim)}{\psi}_\eta(x,\tau)$ to the Langevin equations for fermions or, equivalently, as a quantum average of a functional of the superfields $\overset{(\sim)}{\Psi}(z)$. In the superspace formulation (12) the origin of the SQS chiral anomaly (26) can be understood along the lines of the Vergeles–Fujikawa method [12] as non-invariance of the fermionic superfield functional measure in the generating functional (12a) with respect to (infinitesimal) U(n)-chiral change of variables:

$$\Psi(z)\to\exp[\mathrm{i}\alpha(z)\gamma^{(D+1)}]\,\Psi(z),\quad \tilde{\Psi}(z)\to\tilde{\Psi}(z)\exp[\mathrm{i}\alpha(z)\gamma^{(D+1)}],$$

$$\mathcal{D}\Psi\,\mathcal{D}\tilde{\Psi}\to\mathcal{D}\Psi\,\mathcal{D}\tilde{\Psi}\exp\left(\int\mathrm{d}^{D+1|2}z\,\alpha^a(z)\,\mathcal{A}^a(x)\right).$$

To recapitulate, we have formulated a general scheme for constructing stochastic Noether conserved currents based on the superspace approach [3] to SQS. These stochastic currents reduce to the ordinary Noether currents in the equilibrium limit [cf. (25), (26)]. In particular, the correct chiral anomalies (26) are reproduced from the classically conserved (for $m = 0$) SQS chiral supercurrent (16) and the mechanism responsible for this is understood.

After completion of our manuscript we learned that M.B. Gavela and N. Parga have independently treated the problem of stochastic Noether currents in a different approach starting from Gozzi's functional integral formulation of SQS [13].

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